SOLVING STRATEGIES OF PROBABILITY WORD PROBLEMS AMONG COLLEGE STUDENTS

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ABSTRACT

Various solving strategies in probability of events could complicate the students in choosing the best strategy in problem solutions. This research was carried out to identify the favourite solving strategies used by college students to solve probability and event problems. As a preliminary of a case study, only three students were selected as participants. In this study, the participants have learnt the probability topic in secondary schools as they must apply the knowledge in college level. Think aloud analysis was used to evaluate the influence thought and behaviour of the participants when solving the task. An in-depth semi-structured interview was conducted together with the think-aloud analysis to gain rich and thick descriptions from the participants. Protocols were coded and analyzed to determine the strategies used and the reason of choosing those strategies to solve the problems. Results indicated that participants loved to use tree diagram as solving strategies in solving probability problems without knowing the accuracy of its effect. Therefore, educators can plan strategies, set problems, or prepare activities for students to understand the probabilistic situation and probabilistic reasoning to solve problems. Otherwise the students are merely copying procedures presented in class.

Keywords: solving strategy, problem solving, probability, think-aloud

INTRODUCTION

Probability is a rather difficult concept. When asked about the probability of occurrence, many students perceive that they are asked to predict whether the event will occur, rather than measuring how likely, or often it will occur. Probability is not about predicting whether a particular event will take place but about determining how the probability is propagated over the possible events (Baltaci & Evran, 2016; Beitzel, Stally, & DuBois, 2011; Galavotti, 2015). Besides students, adults also face problems to think about probability and having a disability to solve probability problems. If this continues, students cannot control the probability involving daily situation of decisions and judgments, for example when assessing risk (Batanero, Chernoff, Engel, Lee, & Sánchez, 2016).

Although the topic of probability is being included into the mathematical curriculum, it is still not considered a necessary topic and is considered a topic with least importance and taught as the final topic. Continuous research has given importance on the requirements of probability not only in mathematics but also in physical, biological, and social sciences (Jones, 2005). Now, the probability requirements are emphasized in mathematics and curriculum by introducing several concepts of probability across school curriculum (NCTM, 2010).

Probability is a mathematical branch with importance in science, business and industry research, politics and everyday life (Bamberger, 2003). It is a subject that must be learned by all science and technology students, social sciences and businesses at Matriculation College in Malaysia (Matriculation Division, 2018). Feedback by some mathematics lecturers at the matriculation colleges indicate that although secondary students have learned the probability topics for two years at the secondary level, they are still unable to solve the probability of event properly and correctly. According to the mathematics lecturers, many students did not know the use of the probability solving strategies previously learned. This statement is supported by Danisman and Tanisli (2017) stating that most of the lecturers think that students should be aware of the basic concepts of probability because the introduction to the probabilities has been taught to the students, but due to lack of knowledge of vertical curriculum, lecturers cannot ensure the students' existing knowledge about probabilities.

Extensive exploration of the concept of probability in the curriculum requires further research on the probability of thinking among students of all ages. Although there are studies on student thinking in probability (Ang & Shahrill, 2014; Baltaci & Evran, 2016; Batanero & Sanchez, 2013; Borovcnik, 2012) not many other research has been conducted on student thinking in the classroom (Diaz & Batanero, 2009; Tan & Tan, 2015), and there are a lack of research focused on learning and teaching of probabilities at college level (Corter & Zahner, 2007; Effandi & Yusoff, 2009).

Focusing on the concept of probabilities and events in the classroom, researchers explore the strategies used by matriculation college students in solving probability problems with various forms of settlement. Hence, a study should be implemented in order to contribute to the improvement of the quality of teaching and learning of mathematics in Malaysia.

This study aims to explore the solving strategies used by matriculation college students to solve probability word problems with various forms of solutions. Therefore, the researchers aimed to ascertain the methods employed by the participants in solving probability word problems and the reason to employ these strategies for the given problems. This level of students needs to apply existing knowledge of probability thinking learnt at the secondary level.

METHODOLOGY

Participants. This research study took place at one of the matriculation colleges in Peninsula Malaysia. Approval from Matriculation Centre was received prior to commencing data collection. This study used a qualitative approach, which utilized the case study method. The participants volunteered to participate and provide rich, thick, and in-depth descriptions. The participants are two Malay females and an Indian male with different academic variations. Each participant was compensated with a gift for agreeing to partake in this study.

Procedure. The researchers gave the participants one task with seven probability of event problems. Participants were asked to read each probability problem aloud before beginning the problem and to verbalize their thinking while attempting to solve the problems. The research focused on the solving strategies used by students while dealing with the ambiguity of open-ended problems which may have multiple possible solutions, although some may be more suitable than others.

Think aloud enables the researcher to identify the types of solving strategies used by the students and how the information is acquired and used during problem solving activities. Participants were encouraged to constantly speak aloud the solutions to the problems were developed and their thoughts were described regardless of how insignificant to the strategies are perceived towards the process of solving problems. Participants should reason aloud and verbalize every thought that comes to mind during the problem-solving process. The researcher acted as the interviewer and observer can also prompt participants when adequate details are not provided or during the period of silence.

This method of data collection was recorded on the scene. Video recordings were made of each participant using camera where the behaviour or gesture of the participants were observed during the

interview. Thus, the data provided the student's verbal and non-verbal response simultaneously as they solved the probability word problems.

Coding. Coding for this research was based on the solutions of the problems practically and verbally by the participants when answering the tasks. All tapes were transcribed verbatim. Therefore, the analysis was from the interview transcripts and matched the written solution in solving probability word problems.

RESULTS

This preliminary study shows that there are three methods used by participants during their task-based questions. The dominant methods used are listing and constructing a diagram. The participants decided to use the listing and constructing Venn diagram to solve probability event problems, while tree diagram to solve independent event and conditional probability problems.

From this study, it reveals that the participants create their own solving strategies to solve each type of probability word problems. The solutions given by them shows the logical procedures where they use the information to create the best solution they think. Even the types of solving strategies were same, the solutions were not the same as their probabilistic thinking were different and different students' ways of thinking support the students to use problem solving strategies variously (Intaros, Inprasitha, & Srisawadi, 2014).

Listing

Although the method used is very simple, by only listing the information provided, the students are unable to solve the problems properly. Analysis of think aloud indicates that they understand the question and can state the method to list correctly, but the information released into the sample set is incorrect. Even the mathematical procedure used was correct but the final answers were incorrect.

For Sara, using listing method is beneficial for her as she gets the correct answer for a Dice Problem in getting the probability of one even and one odd number from the two dice that rolled at the same time. She knows the best method to use where she responses directly after reading the questions and solves the problem.

"One even number. One odd number. One two, one three, one five. Then the probability for even number and an odd number is three over six. So, one over two."

However, Susen and Fazi also used listing method to solve the same problems but failed. This shows that the three participants used the same method to solve the problem but the know-how on solving the problem varied. Therefore, the students may need to enhance their knowledge albeit knowing the method to be used.

The set of numbers selection was done using the listing method by the participants in a Wheel of Fortune Problem. The participants need to find the probability which can get a number, which is either a perfect square or the sum of its digits is 7. To solve this problem, they need to know the mathematical concept prior conducting the process. Here are one of the participant's responses and her work.

Student's response	Student's work
"Two square equal to 4. Three square equal to nine. Four square equal to sixteen. Six square equal to thirty-six. Five square equal to twenty-five. Seven square… no lah.	4, 9, 16, 25, 36
So perfect square numbers have five over forty.	5/40
The probability is P (PS) equal to five over forty.	P (PS) = 5/40
Then for SD seven… seven, sixteen, twenty-five, thirty-four.	SD7 = 7, 16, 25, 34
So, probability, P, SD Seven is four over forty.	P (SD7) = 4/40
Briefly, this is one over eight and this one, one over ten.	P (PS) = 1/8, P (SD7) = 1/10
So, for this, the formula is, P (PS) and P (SD7) minus P PS intersect SD7, so one over eight plus one over ten minus one over twenty.	P(PS ∪ SD7) = P(PS) + P(SD7) - P(PS ∩ SD7) = 1/8 + 1/10 – 1/20
The answer is seven over forty." (Fazi)	= 7/40

However, matching the responses and the work, the perfect square listing process is incomplete. The participants did not list the correct numbers in set notation and missed one numbers in the listing. According to the participants, they selected listing strategy because the numbers given were small, that is from 1 to 40, and the numbers can be list out on paper by calculating mentally without using calculators. They can also state the process of calculation or steps taken in solving the problem.

Venn Diagram

The election for Venn diagram as a solution for probability event problem is perceived as the right choice for Newspaper Problem in identifying number of students that did not buy either of the two newspapers from their school store. This illustrates that the students understand the keywords found in the problem. This strategy used gives a positive impact on the students by helping to develop a better and more permanent concept image (Gulkilik & Arikan, 2012; Kurnaz & Arslan, 2014). All the participants gave responses on choosing this method to solve the question, although there are other methods that can be used as a solution.

"And then thirty students bought both NST and Star." (Susen)
"Who buys both is thirty." (Sara)
"If we make a Venn diagram near the same section there are thirty people." (Fazi)

However, all three participants obtained the wrong answer when entering the procedural path. This study focused on the strategy or method used in solving probability word problem only, therefore, the misconception or right and wrong answers were not focused upon.

Tree Diagram

For the question of the independent event type, the method selected by the participants is by using a tree diagram. The selection of the tree diagram method helps the participants to perform well, which in turn contributes to the correct answer. The use of the tree diagram method can help them to solve problems successfully.

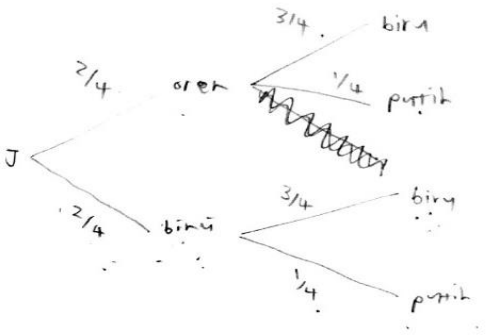
Responses from the participants show that the choosing tree diagram can give them a direct answer to the Spinner Problem in getting the same colour on two spinners. This visual representation is often

helpful in solving problems (Debrenti, 2015; Kurnaz & Arslan, 2014). The participants responded when they constructed the tree diagram.

"Nine is the possible outcome. So, one outcome divides by nine possible outcomes (finding the value on the tree diagram he draws). So, probability to get the same colour is one over nine plus one over nine plus one over nine equal to three over nine." (Susen)

For this type of question, the procedure used after constructing a tree diagram was explained successfully. This situation explains the questions in parallel with the choice of the right solution method employed. The daily situation of problems makes the participant to choose the strategy spontaneously after reading the instructions without having to think about another strategy or hesitate to choose another strategy in a mathematics classroom.

Basketball Problem is a daily life situation that was given to the participants to solve the problem of a young boy who wants to choose his jersey and shorts, both in blue colours. Real life situation also guides the participants to represent the problems in the mathematical form. The chosen strategy shows that visualization really helps them to organize the data and helps them in the calculation. Below is the feedback of a participant matching with his work.

Student's response	Student's work
"There are four jerseys. Two orange and two blue. Break the jersey to two. Orange and blue. So, to get the probability two orange is two over four. So, the probability for blue is also two over four. So, we want to match the jersey with the short pants. The short pants also have two colours. Blue and white. Blue, three over four and white, one over four. So, for blue jersey, two over four and for blue pants three over four. So, we can get three over eight. "(Fazi)	

Conditional probability problem is one of the hardest tasks that are apparently more complex than others (Huerta, 2016). In this study, the selection of tree diagrams for problem solving is basically helpful to the students but if they do not know how to perform the calculations, they would not be able to succeed.

Based on the feedback, the participants was able to draw and label the tree diagram very well, but only one participant knew how to use the labels he has drawn to perform calculations correctly and accurately.

"Given the rain and no rain depicts this, then departs and does not depart for rain and does not rain (point to tree diagram drawn). So, the probability of rain is zero point three so one minus zero point three equals to zero points seven for which does not rain. Departure is zero points one and not departing during rain is zero point nine. Depart on time is zero point eight for no rain, so depart not on time at no rain is zero point two. What is the probability of rain as the flight departs just in time? Ohh ... rain and depart on time (write formula on paper; P (R | TO) = P (R∩TO) / P (TO)). So, R intersect departs on time, zero-point one divide zero point three times zero point one plus zero point seven times zero point eight. So, zero point one over zero point five nine equals to ten over fifty-nine." (Susen)

"Starting to say he has a chance of thirty per cent going to rain, so the probability of rain is zero point three. So, if we want to find, do not rain, one minus zero point three can be zero points seven. Then, the question tells us, if the flight departs on time, so his opponent is not on time. So, the probability of rain is broken into two directions (look at sketches on paper), one for on time and one for not on time. If on time there are zero points one and if not on time, one minus zero points one can be zero points

nine. So, if it does not rain, we break into two. Not rain but on time and one does not rain but he is not on time. So not to rain and on time has given that zero point eight so we minus to get on time that is one minus zero point eight can be zero points two, not to rain and not on time. So, the answer is, zero point zero one five for the probability of rain, the flight departs on time." (Fazi)

From the responses, participants knew how to read from the diagram they constructed, but the participants were unable to calculate the procedural mathematical forms. Incorrect values on each branch of the tree diagram may lead to perform incorrect procedure.

Overall, students loved to imagine and use a tree diagram in all sorts of probability problems. However, as for this question, the participants did not realize that conditional probability was different from other types of probability problems. Students need to memorize the formula for conditional probability even if they know how to sketch and label the branches of the tree diagram correctly.

DISCUSSION

The findings tell us that students loved to choose graphical method (tree diagram and Venn diagram) in their solutions to solve probability word problems. A basic diagram such as tree diagram and Venn diagram have the advantage of scientific and mathematical tasks. This notion is also approved by Beitzel, Staley, and DuBois (2011) in their research where they found out tree diagram and Venn diagram were effective as an aid in solving problems. They also agreed that the construction of tree diagrams makes the difficult task more manageable.

Based on the think aloud analysis, other findings also show that students have no problems with the word problems. Form their responses, the participants seem understand the problems where they analyse the data to make the solution. This is due to the use of mathematical terms presented verbally and in written form. This result contrasted with the research from Boonen, Van Wesel, Jolles, and Van der Schoot (2014) where students faced problems in text comprehension.

The solving strategies chosen by the three participants were coincidently the same for all types of problems given. This also shows that their previous probability knowledge from secondary school were strong but they still cannot identify their cognitive whether it is because of the routine or non-routine problems. Therefore, further research is needed in order to enhance the number of participant and research question to gain more data about the solving strategies used by college students.

Qualitative research was not widely used in mathematics education. Due to this matter, this research will enhance the empirical research in mathematics using qualitative approach. This is because the use of clinical interviews or think aloud analysis is very helpful in understanding students' action and thinking more in depth (Heng & Sudarshan, 2013). The triangulation data improves the credibility of this research.

CONCLUSION

From this study, the participants was interested in using visual diagrams such as tree diagram and Venn diagrams to represent the information given to solve the problem as well as to find the value in conducting the calculation in obtaining the answers. They used diagrams in almost all the probabilities problem in order to get solutions. This further prove that visualization from imagery can help students solve problems even the student is not a visual person.

It can be concluded that although students are unable to conceptualise and do not understand the purpose of the question, students are more likely to choose strategies using diagrams such as the tree diagram and Venn diagram because students presume that they are able to draw mentally prior to making any calculations. This was proved by the participants whereby their solutions in the probability word problem task given were only twenty nine percent correct.

Instead, this preliminary study indicated that the choice of strategies depends on many factors. However, the possible factors or obstacles encountered by that students while solving probability word problems are yet to be identified. There is no evidence that a particular strategy is more superior. This finding creates more synergies to the excitement in solving probability word problems for lecturers or practitioners to improve teaching and learning on probability of an event.

To overcome this setback, the next study will justify the obstacles faced by these college students when solving probability word problems studied over two to four years prior furthering their studies at the undergraduate level. This study will be supported by participant observation and document analysis. Besides, the increasing participation in the study may overcome more data that will be identified clearly in determining the solving strategies used in solving probability word problems among college students.

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