A Pre-Service Mathematics Teacher’s Subject Matter Knowledge of The Mode: A Case Study


ABSTRACT

In this globalized world, our citizens gain instantaneous access to statistical information via various communication technologies. This has heightened a serious call for the development of statistically literate citizens. Nevertheless, research has highlighted that our school leavers are not very statistically literate which drew the attention to teachers and their knowledge in the enhancement of statistical literacy. This paper presents the analysis of the responses of a case study, named Lina (a pseudonym) related to two tasks on the idea of mode. Data was collected using clinical interview based on open-ended questions as instrument. The findings suggest that Lina could relate the idea of mode to real-life situation. Lina utilised the mode as a form of data representation in certain situations. However, Lina did not utilise the mode as a quick method to report an average. In relation to statistical literacy, the understanding of the mode as a form of data representation can lead to this measure being used as a quick method to report central tendency or average.

Keywords: subject matter knowledge, statistical literacy, mode

INTRODUCTION

The last few decades have highlighted the importance of statistic education that focused on two main goals which are: to prepare all students to use statistics in their daily life and to prepare some students with adequate foundation for further formal study of statistics (Watson, 2006).

However, Garfield and Ben-Zvi (2007) pointed out that there exist an important issue concerning school graduates not being able to relate statistics effectively in their daily lives. The issue is found to be alarming because it is believed that students acquire all their statistical literacy skills as part of their school experiences (Garfield & Ben-Zvi, 2004). They are supposed to graduate from schools and enter the society prepared with these skills.

Therefore, this had increased the attention on the development of statistical literacy in schools (Gal, 2002). Watson (2006) argued that, “It is my view that teachers are the next big frontier in bringing statistical literacy to all students to prepare them to leave school and enter society” (p. 271). She firmly believed that teachers should offer their students with productive experiences using real-world examples which demonstrate the utility of statistical concepts in daily life. However, the question now is whether our teachers have the knowledge to enhance statistical literacy in their teachings because “a teacher cannot teach something that she does not know how to” (Wun, 2010; p. 1).

On the other hand, although pre-service mathematics teachers’ subject matter knowledge undergoes transformation when they come into teaching but these teachers do need adequate background
about their subject prior to their induction to teaching upon which greater subject matter competence can be built (Grossman, Wilson, & Shulman, 1989). This also applies to the subject matter knowledge involved in the enhancement of statistical literacy.

Several statistics topics such as probability, inference, variation, and measures of central tendency constitute the subject matter knowledge involved in the enhancement of statistical literacy. However, among all these topics, the topic on measures of central tendency was not only found to be prominent in most mathematics curriculum but also quite dominant in relation to statistical literacy because of its frequent appearances in everyday life (Jacobbe & Carvalho, 2011). This includes Malaysia. Therefore, emphasizing on the need for an adequate subject matter knowledge of this topic among our pre-service mathematics teachers. However, do our pre-service mathematics teachers have the subject matter knowledge of measures of central tendency for the enhancement of statistical literacy?

**Subject matter knowledge** - the subject matter knowledge of beginning teachers involves four dimensions: content knowledge, substantive knowledge, syntactic knowledge, and beliefs about the subject matter (Grossman, Wilson, & Shulman, 1989). Shulman (1986) pointed out that teachers’ understanding of the subject matter should cover not only knowing “that” but also knowing “why”. Shulman’s definition of subject matter knowledge was further widened by Ball, Hill, and Bass (2005) as “mathematical knowledge for teaching” which focuses on the content teachers need to know and what ways they need to know this content for teaching. Hence, in this study, the subject matter knowledge of measures of central tendency was conceptualized as knowledge of the content and organization of the topic measures of central tendency involved in the enhancement of statistical literacy.

**Statistical literacy** - there are many interpretations given to statistical literacy. Wallman (1993) interpreted that statistical literacy as the ability to understand and critically evaluate statistical information along with the ability to appreciate statistical thinking in various life decisions. Similarly, Gal (2002) elaborated statistical literacy based on two components. The first component is the ability to interpret and critically evaluate statistical information, and the second component is the ability to discuss or communicate their reactions to the statistical information. Statistical literacy is also interpreted as a form of connectivity and skills. For instance, Watson (2006) views statistical literacy as the meeting point of the statistics curriculum and the daily world. Whereas, Garfield and Ben-Zvi (2004) view statistical literacy as a set of skills that students should acquire from any statistics program. However, in the present study, statistical literacy is interpreted as the ability to relate central tendency concepts to real-life situations.

The investigation of the subject matter knowledge of measures of central tendency in the present study begins with Gal’s model of statistical literacy which illustrates on the subject matter knowledge components involved in the enhancement of statistical literacy. The model draws upon that the subject matter knowledge involved in the enhancement of statistical literacy includes two major components; knowledge and dispositional components. The knowledge component consists of five cognitive elements: literacy skills, statistical knowledge, mathematical knowledge, context knowledge and critical questions whereas the dispositional component consists of two elements: critical stance and belief and attitudes.

However, in the present study the investigation of the subject matter knowledge of measures of central tendency is focused at Gal’s statistical knowledge of measures of central tendency which includes measures of central tendency and data, familiarity with basic terms and ideas related to measures of central tendency and conclusions reached based on the measures of central tendency (Gal, 2002, p. 10).

This investigation is carried out on four different constructs, namely measures of central tendency with reference to context, measures of central tendency in handling bias, measures of central tendency and problem solving, and measures of central tendency in making inference. Watson (2006) highlighted that these constructs involving the measures of central tendency are important for the enhancement of statistical literacy. The following diagram illustrates on the conceptual framework of the present study:
Therefore, the present study seeks to find the answers to the central question of this research: What types of subject matter knowledge of measures of central tendency do pre-service mathematics teachers have?

However, the present paper reports only on one part of the larger study which is focused on the concept of the mode. The mode is referred to the most frequent value in a data set and is obtained based on the highest frequency. One important characteristic of the mode that distinguishes it from the other measures of central tendency is that it is not necessarily numerical all the time. The mode can be categorical too. However, among the three measures of central tendency, the mode has not drawn a great deal of attention in the literature and occupies a less prominent position in the mathematics curriculum. One reason for this may be due to the perception that the procedure involved in obtaining the mode is far less difficult than the mean or median (Groth & Bergner, 2006).

**METHODOLOGY**

In the present study, a case study research design was employed to investigate and understand, in-depth, pre-service mathematics teachers’ subject matter knowledge of measures of central tendency. “A case study design is used to gain an in-depth understanding of the situation and meaning for those involved” (Merriam, 1998, p. 19). Several researchers (e.g., Chew, 2007; Sharifah Norul Akmar, 1997; Wun, 2010) have employed the case study design to study Malaysian students, pre-service teachers, and lecturers.

The purposeful sampling was used to select the subjects of this study. This paper reports on one case study of a pre-service mathematics teacher named Lina. The subject was given a pseudonym in order to protect the anonymity of the interviewee. Lina is one of the subjects of a larger study involving six pre-service mathematics teachers from a local university enrolled in a 4-year Bachelor of Science with Education (B.Sc.Ed.) program who majored or minored in Mathematics. Lina was 24 years and 1 month old when she was interviewed. Currently, she is pursuing her final semester of a 4-year Bachelor of Science with Education (B.Sc.Ed.) program at a local university. Lina majored in Mathematics and minored in Chemistry and her
current Cumulative Grade Point Average (CGPA) is 3.62. Lina does not have any teaching experience prior to this interview.

A total of fifteen tasks were devised for this study. This paper reports on the responses of the subject on two tasks: Task 2 and Task 3 given in Appendix A. Both the tasks were used to explore the subject’s subject matter knowledge of the mode with reference to context. Task 2 was developed by the researcher and is used to explore the subject’s knowledge of utilising the idea of mode in daily life situation. The task is also used to explore the subject’s knowledge of the mode that represents a typical behaviour of the data. This characteristic of the mode can be used as a quick method for reporting central tendency or average.

Whereas, Task 3 was adapted from a previous study (Konold & Garfield, 1992). Task 3 is used to explore how the subject identifies the mode and to further explore the subject’s knowledge of the mode in representing a typical behaviour of the data. This characteristic of the mode can be used as a quick method of reporting central tendency or average. What appears to be different here is that Task 3 is presented in a different representational form which is tabular as compared to Task 2 which is presented in the form of raw data.

Data for this study was collected using clinical interview techniques based on open-ended questions as instrument. The interview was conducted in the Discussion Room in the Main Library. The physical setting was arranged with a table and two chairs. The interview was recorded using audio voice recorder and a digital video camera. Blank papers, pencils and a calculator were accessible to the subject throughout the interview. Materials collected for the data analysis consisted of the audio recordings, video recordings, subject’s notes, and the researchers’ notes during the interview.

The data analysis process consisted of three levels. The first level, both the audio and video recordings of the clinical interviews were verbatim transcribed into a written form. The transcription included the interaction between the researcher and the subject during the interviews as well as the subject’s nonverbal behaviors. At the second level, the raw data in the forms of transcription were coded, categorized, and analysed according to the specific themes to produce protocol related to the description of the subject’s subject matter knowledge of the mode. Finally at the third level, case study for the subject were constructed based on information from the written protocol where the analysis was carried out to describe the subject’s behaviors in solving the task.

FINDINGS

This section here, will discuss Lina’s knowledge of the mode. In general Lina could utilise the idea of mode in daily life situation.

Utilising the Idea of Mode in Daily Life Situation

When Lina was given a task on the use of the mode in daily life situation without the task explicitly stating so, Lina had successfully utilised the idea of mode in daily life situation. In Task 2 (given in Appendix A), when Lina was asked how she would place the order for the shoe sizes. Lina mentioned that she would place the order for the shoe sizes in her shop by looking at the stock and the frequent size that the customers wore. Lina mentioned the word “frequent” which pointed out to frequency tally, a prerequisite knowledge required for the mode identification. In addition, when she was probed further on how she would do that, she explained that she would base her order on the shoe size that is purchased the most. Her statement “purchased the most” indicated that she had utilised the “most frequent” involved in the idea of mode as shown in Excerpt 1 (S2, Int. 1, 73-78).

Excerpt 1
R: How would you place the order for the shoe sizes in your shop?
S: Depending on the stock and the frequent size that the customers wore.
R: How do you do that?
S: I will check the stock and then order based on the shoe size that is purchased the most and the rest follows.
Mode as a Form of Data Representation

When Lina was asked to choose one female shoe size, Lina mentioned that she would choose size five. Her decision was based on the mode. This was further reconfirmed, when she was probed on why she would choose that particular shoe size. Here Lina mentioned that the shoe size is favoured by her customers. The word “favoured” showed “the most” which indicated the mode. After then, Lina was asked if she would use the chosen shoe size to represent the female shoe size in the shop. Lina mentioned that she would use the chosen shoe size to represent the shoe size in her shop because “majority” bought that particular shoe size. Lina’s decision on utilising the chosen shoe size to represent the female shoe size in her shop and justifying the reason because “majority bought” that particular shoe size indicated that Lina had utilised the mode as a form of data representation. The following Excerpt 2 (S2, Int. 1, 79-86) illustrates this point:

Excerpt 2
R: If you want to choose one female shoe size, which shoe size would you choose?
S: five
R: Why?
S: Because... according to this data, there are many customers who favour size five compared to the rest.
R: Can the chosen shoe size represent the female shoe size in the shop?
S: Can because the majority bought that.

Utilising the Mode as the Average

However, when Lina was asked on what she would use as the average shoe size. Lina mentioned five. When she was probed to explain her decision, she mentioned that her decision was based on two answers which are “majority” and also from the formula which she got five point something as described in Excerpt 3 (S2, Int. 1, 87-91). The word “majority” showed that one of Lina’s answer was on the idea of the mode whereas the statement “the formula we get five point something” indicated that she had used some sort of algorithm here.

Excerpt 3
R: What would you use as the average shoe size?
S: five
R: Why?
S: There are two answers because of the majority and because of the formula we get five point something. So it is five.

However, when the researcher probed her further with the question: “what is the average?” Lina mentioned “five point something” but she said that she did not calculate. Lina asked the researcher if it needed to be calculated. The researcher mentioned if she felt she could explain better if she calculated then she should. In which later, she performed the calculation and obtained the value five point four. Lina’s calculation confirmed that “the formula” that Lina was referring to earlier was the mean algorithm as shown in Diagram 1.
Based on Lina’s response, the researcher probed her further by repeating again the question: “what would you use as the average shoe size?” Again, Lina mentioned that it would be five and justified her decision based on the “majority” bought that size and because of the “average five point four”. Her response again indicated that Lina was utilising the idea of mode from the word “majority”. However, Lina was also utilising the mean from the statement “the average five point four”. Therefore, Lina did not only utilise the mode but also mean for the average shoe size. The following Excerpt 4 (S2, Int. 1, 104-111) points out this:

Excerpt 4

R: What is the average here?
S: five point. I did not calculate. Five point something. Should I calculate?
R: Maybe, if you can explain better if you have calculated the value.
S: five point four
R: What would you use as the average shoe size?
S: Size five
R: Why?
S: Because majority bought that size and is because of the average five point four.

Context of the Data

The researcher delved further and asked Lina: what does average relate to in terms of measures of central tendency. Lina mentioned the mode and the mean. However, when Lina was probed further, she expressed that the average is actually the mean. Lina clearly distinguished that the “average is actually the mean” and “the mode is the most frequent”. Nevertheless, when Lina was asked again on her decision on the average shoe size, she still persistently mentioned “five” which she mentioned that she based on the “most frequent” and “the formula”. Her justification, emphasised again on both the idea of the mode and the mean in which made the researcher to probe Lina further and clarify if “the most frequent” would it indicate the mode.

Here Lina showed some confusion initially. She was assured by the researcher not to worry but to explain her answer. After which she firmly mentioned that the average is actually the mean. The researcher wanted Lina to explain her answers to the questions on what she would use as the average and what is the average? Lina had given two different answers to these questions where the question on what she would use as the average, she mentioned five and for the question on what is the average, she mentioned five point four. Here Lina clearly explained that her decision to use “five” as the average shoe size was because she knew that the shoe size has to be presented as a whole number and that “most frequent” was used to decide on which whole number is suitable to be used as the average. Her explanation clearly indicated that Lina knew that for the context of the given data, the ideal average has to be a whole number because shoe sizes are mostly in whole numbers. Therefore, it was also clear here that Lina’s knowledge of average is mean
when she expressed that “average is actually the mean”. Lina was only using the mode to guide her decision on the suitable average value based on the context of the given data. Lina did not utilise the mode as the average. The following *Excerpt 5* (S2, Int. 1, 120-138) illustrates this:

**Excerpt 5**

R: What does average relate to in terms of the measures of central tendency?
S: Mode and mean
R: So are you telling that average can be associated to both mode and mean?
S: Average is actually the mean
R: But earlier on, you mentioned that it can also be related to the mode
S: No no no... average is actually the mean. Mode is the most frequent.
R: If I asked, what would you use as the average shoe size?
S: Size five
R: How did you get this?
S: Using most frequent and the formula
R: So if the most frequent, don’t you think it is indicating the mode.
S: I am checking my answer. I think I screw up everything. (Looking confused)
R: Don’t worry. Just explain to me your answer.
S: Average is actually the mean.
R: But when I asked you what you would use for average, you mentioned five and when I asked you what the average is, you gave me five point four. Can you explain this?
S: Yes. The average that I calculated is five point four. But in this case, the average, if it is not going to be five it is going to be four. We have to use a whole number because shoe size. I used the “most frequent” to decide on my average.

**Mode Based on the “Highest Number” and “Most Frequent”**

After finishing with Task 2, Lina was given Task 3 (given in Appendix A). It was found that from this task, Lina had two different ways of identifying the mode. She initially identified the mode based on the “highest number” but later changed her answer to the “most frequent”.

When Lina was given Task 3, she was asked to identify the mode. Here Lina quickly mentioned that the mode is student D. When Lina was asked on how she identified the mode. She explained that she based her decision on the highest number of comments. Later, the researcher, changed the value 22 for the number of comments in the table to 6. Lina was asked again on what is the mode now. In which she mentioned that the mode is still student D. When the researcher asked her to explain on how she obtained the mode. Lina again mentioned that she based her answer on the highest number of comments.

However, after a few seconds, Lina showed some uncertainty and was thinking for a while. She realized that her answers earlier was incorrect after which she requested to change her answer. Lina changed her answer to the mode “two” which she explained that was based on three students who made two comments. Lina’s statement on “the mode is two” because three students who made two comments” indicated that now Lina had based the mode of the “most frequent”. The following *Excerpt 6* (S2, Int. 1, 147-158) illustrates Lina’s decision on the mode:
Excerpt 6
R: What is the mode?
S: It is student D.
R: How did you obtain this value as the mode?
S: Based on the highest number of comments
R: If I change this value, 22 to 6. What is the mode now?
S: Still the same which is student D.
R: How did you obtain the mode?
S: Based on the highest number of comments made by the students. Oh sorry, sorry, sorry [she looked uncertain now and was thinking]. (Paused awhile). I want to change my answer.
R: What is your answer?
S: The mode is “two” because there are three students who made two comments.

Mode as a Form of Data Representation

However, when Lina was asked if she would use this value (the mode) to represent the number of comments made by students on that day. She mentioned “no’ straight away. When the researcher probed her “why”. She explained that the mode and the other number of comments had wide differences. She mentioned that the lowest number of comments was zero and the highest number of comments was 6 so the range is six. Lina included the standard deviation and dispersion and mentioned that the standard deviation was roughly three point something. However, here Lina showed uncertainty to her response and requested for some time to think.

After thinking about it for a while, Lina changed her response and explained that the mode “two” can be used to represent the number of comments made by the students on that day. She elaborated that because the mode “two” is not far different than the other number of comments in which she was referring to the case where the number of comments for Student D was 6. However, when she was asked to explain for the case where the number of comments for Student D was 22, Lina explained that the mode “two” cannot be used to represent the number of comments made by the students on that day because of the extreme difference between the mode and certain values given in the number of comments. Her statement here indicated that Lina would only use the mode as a form of data representation when the value is found to be close to the other values in the given data set. The following Excerpt 7 (S2, Int. 1, 171-189) illustrates on this:

Excerpt 7
R: If you were to represent one value for the number of comments made by the students on that day, would you use this value?
S: No
R: Why?
S: Because if we look at the value, the number of comments made by students carry a quite wide difference. The lowest number of comments is zero and the highest is 6. So the range is 6. Looking at in terms of the standard deviation and also the dispersion, the standard deviation is roughly 3 point something. [Looked rather unsure of her answer] Can I think for a while?

R: Can you think a little longer?
S: It can be used. (Refers to the mode)
R: Why?
S: Because the difference between the numbers of comments is not that extreme (for the case where 6 is used) compared to the previous one which is 22.
R: What if for the case of 22?
S: In this case, I would not use this value because the difference is very extreme. One is zero and the other 22.
DISCUSSION AND CONCLUSION

Based on the above findings, it can be concluded that Lina had adequate knowledge regarding utilising the idea of mode in daily life situation. She could relate the idea of mode to real-life situation. She utilised the frequency tally, a pre-requisite knowledge used to obtain the mode in Task 2. Lina successfully obtained the mode based on the “most frequent” in both the tasks despite her having some confusion initially on identifying the mode in Task 3 in which she initially based on the “highest number”. Lina also had the knowledge that the mode is not necessarily numerical all the time but can be categorical too. This is seen when Lina initially identified the mode in Task 3 based on the “highest number” and mentioned the mode as Student D. This finding is in contrast to the findings of Groth and Bergner (2006) which reported that the pre-service elementary teachers in their study mostly implied that the mode is only of use for numerical data sets.

However, in regarding the knowledge of the mode as the average, Lina had limited knowledge. Lina did not utilise the mode as a quick method to report an average. Instead, she only used the mode as a guide to decide on the average in which she actually based on the mean. Lina realized that for the context of the data which was referring to shoe sizes, the ideal average has to be a whole number. Therefore, the mode was used to guide her decision on the appropriate whole number for the ideal average.

This finding is found to be in contrast to Watson (2006) that mentioned that the average can be based on the mode to describe a typical behaviour of a data set. Furthermore, Lina believed that the average is synonymous to the arithmetic mean, which is similar to the finding of Leavy and O’Loughlin (2006) which revealed that almost 52% of the pre-service mathematics teachers’ in their study view the term average as synonymous to the arithmetic mean. Thus, the knowledge of the mode as a measure of what is typical in a data set (Groth & Bergner, 2006) which can be used as a quick method in reporting average is found to be lacking in Lina’s knowledge of mode. Therefore limiting her knowledge in relation to the enhancement of statistical literacy because in regards to statistical literacy, the general term average can mask any of the three measures of central tendency (Watson, 2006).

On the other hand, Lina utilised the mode as a form of data representation. Therefore Lina had the knowledge of mode as a representative of a data set from which they arise (Groth & Bergner, 2006). However, her knowledge of the mode is considered limited because she only utilised this idea to cases where the data values are close to the mode and not for cases where there exist an extreme data value.

Although this paper reports only on Lina’s responses to Task 2 and Task 3 but the researchers will further study her attempts to other tasks related to the mode which will assist the researchers to understand Lina’s subject matter knowledge of the mode. Similar tasks will also be used on other pre-service mathematics teachers in order to explore their subject matter knowledge of the mode. The final findings of this study are wished to provide some substantive results on not only pre-service mathematics teachers’ subject matter knowledge of the mode but also measure of central tendency. It is hoped that these findings can provide some suggestions to the Mathematics teachers education programs at our local universities or teacher training institutes so that a revisit and reconstruct of some fundamental ideas of central tendency concepts important for the enhancement of statistical literacy at a deeper level can be done.

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REFERENCES


APPENDIX A

Task 2

Information below shows female shoe sizes purchased in your shop.

4  5  5  5  6  7  6  6  7  4  4
5  5  5  7  7  7  8  9  5  5  4
6  7  8  9  5  5  6  7  5  4  5

a. How would you place the order for the shoe sizes in your shop?

b. If you want to choose one female shoe size, which shoe size would you choose? Probe: Why?

c. Can the chosen shoe size represent the female shoe size in your shop?
    Probe: Why?

d. What would you use as the average shoe size?
    Probe: Why?

Task 3

The table below shows the number of comments made by eight students during a class period on a particular day.

<table>
<thead>
<tr>
<th>Student</th>
<th>Number of comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>D</td>
<td>22</td>
</tr>
<tr>
<td>E</td>
<td>3</td>
</tr>
<tr>
<td>F</td>
<td>2</td>
</tr>
<tr>
<td>G</td>
<td>1</td>
</tr>
<tr>
<td>H</td>
<td>2</td>
</tr>
</tbody>
</table>

a. What is the mode?

b. How did you obtain this value as the mode?

c. If you were to represent one value for the number of comments made by the students on that day, would you use this value?
    Probe: Why?