

## STUDENTS' ACHIEVEMENT IN SYMMETRY OF TWO DIMENSIONAL SHAPES USING GEOMETER'S SKETCHPAD

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### ABSTRACT

The purpose of this study is to determine the effects of dynamic geometry software Geometer's Sketchpad (GSP) on students' achievement in symmetry of two dimensional shapes. A total of 36 mixed ability Year 3 pupils from a primary school in a rural area of Kedah participated in the study. They were divided into two groups: the experimental group was subjected to lessons shaped with Geometer's Sketchpad (GSP) while the control group was subjected to lessons using the traditional method. The difference between experimental and control groups' achievement was measured using post-test. Students' attitude toward learning of symmetry was measured using a questionnaire with a four-point Likert Scale. Results of the study showed that the experimental group performed better than the control group. These findings indicated that the use of Geometer's Sketchpad (GSP) improve the pupils' achievement and attitude towards learning symmetry of two dimensional shapes.

**Keywords:** *geometer's sketchpad (GSP), dynamic geometry software, symmetry of two dimensional shapes*

### INTRODUCTION

According to Guven and Kosa (2008), geometry is the study of shape and space. Symmetry is an essential part of geometry, shapes and nature (Knuchel, 2004). It creates patterns and organizes our world conceptually. Symmetry is crucial in problem solving because it connects Geometry, Algebra, Probability and other Mathematical Analysis (Leikin, Berman, & Zaslavsky, 2000). Concept of symmetry can be used in translation, rotation which is also known as radial symmetry and reflection which is also known as bilateral symmetry. According to the National Council of Teachers of Mathematics, pupils in grades 3 to 5 should be able to apply the concept of symmetry to analyse mathematical situations (Knuchel, 2004). They should be able to predict and describe the results of sliding, turning and flipping two dimensional shapes. On the other hand, having an understanding of symmetry concept helps to describe the motions that will show that the object and the image are congruent alongside being able to identify the lines of symmetry of a two dimensional shape.

Dynamic geometry software is a powerful tool in enhancing teaching of symmetry and understanding the concept. Van Voorst (1999) stated that exploration and visualization of mathematical concepts using multimedia environment can cultivate a deeper understanding. Students can explore, create and make conjecture about symmetry and its properties on their own using dynamic geometry software.

Geometer's Sketchpad (GSP) is one of the dynamic geometry software for exploring mathematical concepts. This software enables students to create mathematical shapes and interact with them. GSP with exploratory approach helps the students to manipulate, analyse, conjecture and test a concept. GSP software can be used to teach geometry because it promotes learning by testing conjecture on geometrical shapes, relation and transformation (Kurz, Middleton, & Yanik, 2005).

According to Noraini, (as cited in Hock, Tarmizi, Yunus, & Ayub, 2015) Geometer's Sketchpad encourages teachers to have better teaching process, guidance methods and allocate adequate time for the students to explore the programme themselves. Dynamic geometry software such as GSP provides a wide range of opportunity for the learners to explore the visualization, simulation and dynamic while studying geometry (Andraphanova, 2015). Lester (1996) stated that students' attainment in geometry has been improved with the use of GSP while Yousef (1997) mentioned that students' attitude towards learning geometry has been changed.

### **NEED OF STUDY**

Mathematics is an art involving number sense, reasoning and problem solving. Most students do not show high interest towards the learning of mathematics because they get bored with the traditional method of teaching. Therefore, the teaching and learning of mathematics could be improved by using mathematical software. Dynamic geometry software such as Geometer's Sketchpad could boost interest for the students to learn mathematics in a better way.

Idris (2007) stated that individual's cognitive development, mathematical system and teaching methodology could explain the difficulties that faced by primary pupils in learning geometry. Individual cognitive ability is not just about visual perception, but also decision making, which is crucial in order to achieve higher-order thinking in learning geometry (Siew, Chong, & Abdullah, 2013).

Symmetry is one of the topics in the primary mathematics syllabus where students face difficulties in understanding its concept and application in daily life situation. Teachers use the traditional method which is teacher centred in order to teach this topic. When learning symmetry in geometry, students should be able to identify the two dimensional shapes that have the line of symmetry, draw the line of symmetry and use the lines of symmetry to study properties of shapes. In the advanced level, students should be able to identify the two dimensional shapes that have rotational symmetry, determine the angle of rotation and use rotational symmetry to examine properties of shapes (Ohio Department of Education, 2017).

According to Hansen, Drews, Dudgeon, Lawton, and Surtees (2017), children often assume that half of an object is the reflection of the object itself. Moreover, a deficiency of understanding of the distance of the perpendicular from the line of symmetry to any point on the shape is also a proof of identifying the diagonals as lines of symmetry. Some students fail to fulfil the need to maintain the length of the sides in the reflection and the equal distance between points. The inaccuracy of reflecting a shape in the diagonal line of symmetry shows that they lack understanding on the concept where the point of the reflected shape must have equal perpendicular distance from the symmetry line. All these misconceptions lead to negative attitude towards learning symmetry among primary school students.

The use of Geometer's Sketchpad in teaching and learning the topic symmetry provides a platform for the students to create symmetrical geometrical shapes and explore the concept on their own. This study mainly focused on improving students' achievement in symmetry using the Geometer's Sketchpad.

### **OBJECTIVES OF STUDY**

The objectives of this study are:

1. To determine the effects of dynamic geometry software Geometer's Sketchpad (GSP) on students' achievement in symmetry of two dimensional shapes.

2. To investigate the students' attitude towards learning of symmetry of two dimensional shapes using Geometer's Sketchpad (GSP).

This study is also aimed at answering the following research questions:

1. Is there any significant difference in the students' achievement in symmetry of two dimensional shapes between the experimental and control group?
2. What is the students' attitude toward learning of symmetry of two dimensional shapes using Geometer's Sketchpad (GSP)?

### **SIGNIFICANCE OF STUDY**

This study would provide a guideline for utilizing technology for the teaching of mathematics especially in the geometry topic. A dynamic geometry software Geometer's Sketchpad was used in this study. This is one of the dynamic geometry software which can be used in teaching and learning mathematics and the country license was purchased by the Malaysian Ministry of Education in the year 2003. With the intention of catering to individual differences and consistently develop students' cognitive ability, appropriate guidance is needed (Hock et al., 2013). According to Tay (as cited in Abdullah & Zakaria, 2013) manipulative materials can be substituted by dynamic geometry software to give students the opportunity to explore the concepts of geometry.

As a result, the findings of this study will lead teachers to use other resources such as technology to ensure that the students understand a concept and be able to apply it. Tay, Lim, Kim, and Koh (2012) mentioned that the use of technology especially dynamic geometry software could assist students to enhance their performances as well attitude. It is also a platform to facilitate the learning of higher order thinking. Lastly, the findings of this study would be useful for teachers to embark an intervention to improve students' mathematics achievement and attitude towards learning Mathematics.

### **LITERATURE REVIEW**

Integration of dynamic geometry software such as the Geometer's Sketchpad into the teaching of geometry is more effective than the traditional approach in stimulating students' mathematical thinking skills (Idris, 2007). According to Liu and Cummings (2001), Geometer's Sketchpad is a software package designed to enhance children's understanding of principles of geometry. Thompson (2006) stated that the integration of dynamic geometry software enhances students' understanding of plane geometry besides improving their van Hiele levels of geometric thinking. Geometer's Sketchpad (GSP) is a dynamic software that revolutionize mathematics teaching and studying especially in geometry (Kamariah Abu Bakar, Ahmad Fauzi Mohd Ayub, Luan, & Rohani Ahmad Tarmizi. (2010). It can be used for creating, exploring, and testing a mathematical concept in more effective and creative way which leads to a deeper understanding.

Research has shown that teaching geometry using dynamic geometry software Geometer's Sketchpad could enhance students' learning of plane geometry (Choi, 1999). Learning of mathematics with GSP is easier compared to the conventional method (Teoh & Fong, 2005). According to Giamatti (1995) students can explore simple and complex theorems and relations in geometry using the Geometer's Sketchpad. They can also save and record their constructions as scripts in Geometer's Sketchpad for future reference. Geometer's Sketchpad could increase students' achievement in geometry and change their attitude towards learning geometry (Kamariah et al., 2010).

Geometer's Sketchpad is essential for enhancing students' van Hiele levels of geometric thinking because it allows them to investigate geometric concepts and discover relationships among these concepts (Chew & Lim, 2013). Hannafin and Scott (2001) stated that in order to encourage exploratory activities and student-directed inquiry using the Geometer's Sketchpad, a series of structured activities

and guiding questions should be provided to guide students to the point where conjectures are possible. Therefore, students can learn to think critically and become better problem solvers. Moreover, Geometer’s Sketchpad provides learning in a virtual environment. Knuchel (2004) stated that using dynamic geometry software Geometer’s Sketchpad improves higher order thinking skills since students explore and create their own ideas alongside concepts of geometry and symmetry.

**METHODOLOGY**

**Research design**

The research design used for this study was a non- equivalent quasi experimental design using the post-test only. The experimental group was taught using dynamic geometry software Geometer’s Sketchpad while the control group was taught using traditional method. Both experimental group and control group consisted of 18 mixed ability students.

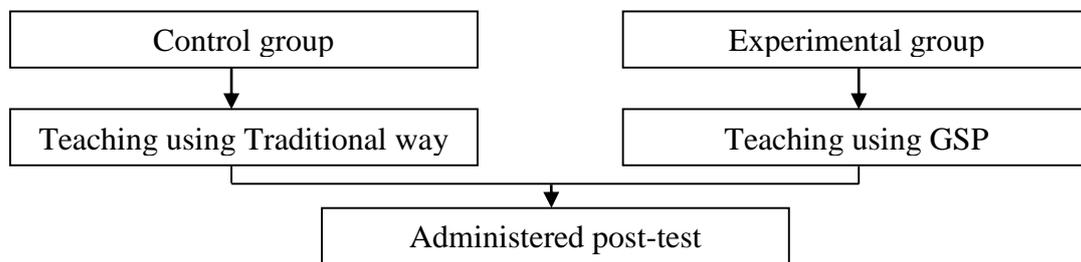


Figure 1. Flow chart of the research process.

**Samples**

The study involved 36 participants who were mixed ability Year 3 pupils in terms of mathematics achievement in the midterm examination of the year from a primary school in a rural area of Kedah, Malaysia. The participants were equally divided into two groups: 1) experimental group which was taught using GSP and 2) control group which was taught using the traditional method. The participants involved were male and female participants.

Table 1

*Participants’ mathematics ability level*

<b>Group</b>	<b>High</b>	<b>Average</b>	<b>Total</b>
Control Group	12	6	18
Experimental Group	10	8	18
<b>Total</b>	<b>22</b>	<b>14</b>	<b>36</b>

**Instruments and Data Collection**

The instruments used for data collection of this study were the post-test and a questionnaire.

a) Post-test

The post-test was used in this study to measure the pupils’ understanding and achievement in symmetry of two dimensional shapes. This test consisted of 12 questions and was devised by the researcher. It used a paper and pencil procedure and pupils were given 30 minutes to complete it.

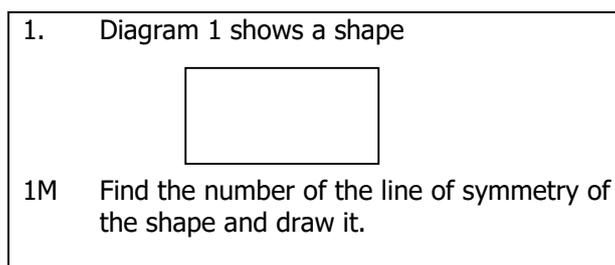


Figure 2. Sample item in post-test

b) Questionnaire

The questionnaire was used in this study to investigate students’ attitude towards learning of symmetry of two dimensional shapes using Geometer’s Sketchpad (GSP). This test consisted of 10 questions with four-point Likert Scale from strongly disagree to strongly agree. Students expressed their agreement or disagreement with the statement to describe their attitude towards learning of symmetry of two dimensional shapes using Geometer’s Sketchpad (GSP).

The validity of post-test and questionnaire was determined by content validity to ensure the content of the instruments are suitable and pertinent to the research. The internal consistency of the questionnaire was checked using the Cronbach alpha value and scored 0.68.

**Procedures**

This study used teaching and learning activities using Geometer’s Sketchpad to help the pupils to understand the concept of symmetry. The pupils were exposed to constructing two dimensional shapes. They also drew the lines of symmetry for the constructed shapes. Pupils explored and discovered the concept of rotation and reflection symmetry.

Table 2  
Summary of instructional activities

Lesson	Instructional Activities
1	<ul style="list-style-type: none"> <li>a) Students open the pre-constructed GSP sketch of points. Students use the segment tool to construct regular polygons in the pre-constructed GSP sketch of points.</li> <li>b) Students edit the constructed two dimension shapes by changing their colour. Students choose polygon interior from the construct menu. To change the colour of the shapes, students choose colour from the display menu. Then, students name the polygons using the text tool.</li> <li>c) Students save their work in the computer.</li> </ul>
2	<ul style="list-style-type: none"> <li>a) Students open their work and construct all the midpoints of the sides of polygons by using construct midpoints tools.</li> <li>b) Students construct the symmetrical lines of through the mid points by using segment tool.</li> <li>c) Students count the symmetrical lines of each polygon and number them using text tool.</li> <li>d) Students explore the rotation symmetry.</li> </ul>
3	<ul style="list-style-type: none"> <li>a) Students use the construct tools to construct other two dimension shapes showed by the teacher.</li> <li>b) Students edit the constructed two dimension shapes by changing their colour. Students choose Polygon Interior from the construct menu. To change the colour of the shapes, students choose colour from the display menu.</li> </ul>

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- 4
    - c) Students save their work in the computer.
    - a) Students open their work.
    - b) Students reflect the constructed two dimension shape using any side of the shape as the mirror line.
    - c) Students use the transform tool to mark mirror and do reflection.
    - d) Students explore the reflection symmetry.
  - 5
    - a) Students use the construct tools to construct other two dimension shapes showed by the teacher.
    - b) Students construct a midpoint of any side of the shape.
    - c) Students then construct a segment from the midpoint to any other point as mark mirror.
    - d) Students construct interior using any three or four points using construct tool.
    - e) Students reflect the constructed interior using the mirror line.
    - f) Students explore the shapes with no symmetrical line.
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**Data Analysis**

SPSS 22 was used to analyse the data of this study. Descriptive statistics was used to analyse the data of post-test and the questionnaire. Independent *t*-test was used to find the significant difference in the students’ achievement in symmetry of two dimensional shapes between the experimental and control group. Frequency table and bar chart were used to analyse students’ attitude towards learning of symmetry of two dimensional shapes using Geometer’s Sketchpad (GSP).

**DISCUSSION OF THE RESULT**

***Research question 1: Is there any significant difference in the students’ achievement in symmetry of two dimensional shapes between the experimental and control group?***

Independent samples *t*-test was carried out to answer the first research question.

Table 3  
*Independent t-test comparing the post-test result*

Group	N	Mean	SD	df	<i>t</i>	Significant
Experimental	18	33.67	2.64	34	8.129	0.000
Control	18	23.17	4.81			

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The *t*-test results was significant,  $t(34) = 8.129, p < .05$ . This shows that a significant difference exists in the students’ achievement in symmetry of two dimensional shapes between the experimental and control group. The mean difference value of 10.5 shows that the students in the experimental group ( $M = 33.67, SD = 2.63$ ) performed better compared to the students in the control group ( $M = 23.17, SD = 4.81$ ). According to Cohen’s (1988) the effect size (2.71) is large since the difference between means exceeds one standard deviation.

***Research question 2: What is the students’ attitude towards learning of symmetry of two dimensional shapes using Geometer’s Sketchpad (GSP)?***

Frequency table and bar chart were used to analyse students’ attitude towards learning of symmetry of two dimensional shapes using Geometer’s Sketchpad (GSP).

The results in Table 4 showed that all the students in experimental group had a positive attitude towards learning with Geometer’s Sketchpad. All 100% of the students strongly agreed that GSP motivates them

to study while about 88.9% of them strongly agreed that the lesson was interesting when GSP was used. About 83.3% of the students strongly mentioned that they had the chance to explore the rotation and reflection symmetry using GSP. However, only 16.7% of them strongly agreed that GSP helps them to understand the concept of symmetry easily. In a nutshell, this study concludes that GSP could enhance positive attitude in the learning of symmetry among the students because GSP could motivate, build students' confidence as well as giving them the opportunities to control their own learning where they can learn independently.

Table 4  
*Results of students' attitude towards GSP*

Items	1 <i>Strongly disagree</i> %	2 <i>Disagree</i> %	3 <i>Agree</i> %	4 <i>Strongly agree</i> %
Lesson in this subject is interesting when use GSP	0 (0)	0 (0)	2 (11.1)	16 (88.9)
I am getting on well in this subject with GSP	0 (0)	0 (0)	7 (38.9)	11 (61.1)
GSP helps me to understand the concept of symmetry easily	0 (0)	0 (0)	15 (83.3)	3 (16.7)
I understand this subject better with the help of GSP	0 (0)	0 (0)	15 (83.3)	3 (16.7)
I got chance to explore the rotation and reflection symmetry using GSP	0 (0)	0 (0)	3 (16.7)	15 (83.3)
I am confident in learning this subject with GSP	0 (0)	0 (0)	2 (11.1)	16 (88.9)
GSP give me opportunities to work with other students in this subject	0 (0)	0 (0)	7 (38.9)	11 (61.1)
GSP give me opportunities to control my own learning where I can learn independently	0 (0)	0 (0)	2 (11.1)	16 (88.9)
GSP motivates me to learn this subject	0 (0)	0 (0)	0 (0)	18 (100)
I dare to explore other mathematical concepts using GSP	0 (0)	0 (0)	4 (22.2)	14 (77.8)

**CONCLUSION**

From the results and discussion, it can be concluded that the dynamic geometry software, Geometer's Sketchpad (GSP) could improve students' achievement in learning symmetry of two dimensional shapes. Geometer's Sketchpad also improves students' positive attitude toward learning of symmetry of two dimensional shapes. Geometer's Sketchpad has been useful as an effective teaching and learning tool in primary Mathematics. The findings of this study have raised implications on the teachers and on the students as well. Teachers could save their time preparing teaching tools by using ICT as a teaching tool. In addition, the use of mathematical software could create more meaningful classroom teaching.

GSP is an effective platform for the students to learn independently by exploring, analysing and collaborating. However, the students should be given more time to experience learning with GSP.

It is recommended that more studies on the application of GSP be carried out in future to promote a better learning style among the students and enhance students' higher order thinking skills.

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